Hard exclusive meson production at next-to-leading order.

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Abstract

We evaluate perturbative next-to-leading order corrections to the hard exclusive leptoproduction of π^+ mesons on a transversely polarized proton target. A model dependent study shows that these corrections can be large. We analyze the scale dependence and explore the Brodsky-Lepage-Mackenzie scale setting procedure. Although the predictions for the cross section suffer from theoretical uncertainties, the transverse nucleon single spin asymmetry turns out to be a rather stable observable since higher order effects approximately cancel there.

Keywords: hard exclusive meson production, next-to-leading order corrections, generalized parton distribution

PACS numbers: 12.38.Bx, 13.60.Le

1 Exclusive meson production and QCD factorization.

The hard exclusive leptoproduction of a meson M from a nucleon target N,

$$\ell(k)N(P_1) \to \ell'(k')N'(P_2)M(q_2) \tag{1}$$

is a promising process to test our understanding of QCD in exclusive reactions, as well as a means for studies of the properties of nucleon to hadron transitions, $N \to N'$, with N' being a baryon from an $SU_f(3)$ multiplet. If the intermediate photon is longitudinally polarized and has a large virtuality $\mathcal{Q}^2 = -q_1^2$, the photoproduction amplitude $\gamma_L N \to N'M$ factorizes into a convolution of three parts [1], see Fig. 1,

$$\mathcal{A} = \sum_{f,f'=u,d,s} \int_0^1 du \int_{-1}^1 dx\, \phi_{ff'}(u|\mu) T_{ff'}(u,x,\xi|\mathcal{Q},\mu) A_{ff'}(x,\xi,\Delta^2|\mu) + \dots, \tag{2}$$

where the ellipsis stand for power suppressed contributions in $1/\mathcal{Q}$. The hard subprocess $T_{ff'}$ encodes the short distance dynamics of the parton scattering and can be consistently calculated in QCD perturbation theory as a series in the strong coupling α_s. The other two blocks, ϕ and A, are universal, i.e. process independent, and embody the long-distance physics. ϕ is a conventional leading twist meson distribution amplitude. It describes the minimal Fock component of the meson wave function with two quarks having momentum fractions u and $1-u$ w.r.t. the meson momentum, respectively. A is a flavour nondiagonal generalized parton distribution (GPD). Its definition, taking apart the flavour transition, differs from the conventional Feynman parton densities by the presence of a non-zero momentum flow, $\Delta = P_2 - P_1 = q_1 - q_2$, in the t-channel. As a result, the GPD is a complicated function of three variables, the s-channel momentum fraction x, its t-channel counterpart[1] $\eta \sim \Delta_+$, and Δ^2. The appearing phase space picture is rich and results into a trinity interpretation of GPDs: they share common properties with forward parton densities, distribution amplitudes, and form factors.

In the present study we will concentrate on the next-to-leading order (NLO) analysis of the π^+-production from the proton. Thus we set $N = p$, $N' = n$ and $M = \pi^+$ in Fig. 1. This reaction is an issue of intensive experimental studies by HERMES [2] and Jefferson Lab [3] in relation to the measurement of the helicity-flip GPDs accessible in this process. Our consequent presentation is organized as follows. In the next section we introduce the definitions of all basic ingredients entering the amplitude (2) and calculate the electroproduction cross section for the transversely polarized proton. In section 3 we present one-loop corrections for the hard-scattering amplitude. In section 4 we introduce simple models for the GPDs and then we discuss the issue of scale

[1] Note that in the kinematics, we are considering, the skewedness parameter η is approximately equal to a (negative) generalized Bjorken variable ξ, to be specified below.

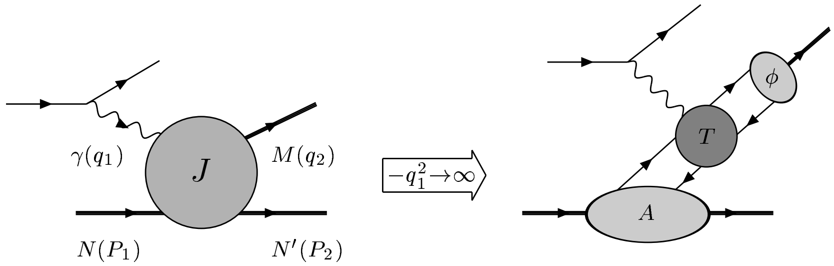

Figure 1: Factorization of the meson leptoproduction at $-q_1^2 \to \infty$ into hard-scattering amplitude T and non-perturbative functions A, the generalized flavour changing parton distribution, and ϕ, the distribution amplitude of the outgoing meson.

setting. After this we give numerical estimates of the differential cross section and transverse single spin asymmetry. Finally, we summarize.

2 Amplitude and cross section for $\gamma_L p \to \pi^+ n$.

The hadronic part of the $\gamma p \to \pi^+ n$ process is given by the Fourier transform of the matrix element of the electromagnetic current $J_\mu = \sqrt{4\pi\alpha_{\rm em}} \sum_i Q_i \bar\psi_i \gamma_\mu \psi_i$

$$\int d^4x \mathrm{e}^{-iq_1\cdot x} \langle \pi^+(q_2) n(P_2) | J_\mu(x) | p(P_1) \rangle = i(2\pi)^4 \delta^{(4)} \left(q_1 + P_1 - q_2 - P_2 \right) \mathcal{A}_\mu^{\pi^+}, \tag{3}$$

with $q_1 = k - k'$ being the difference of incoming and outgoing lepton momenta. Its leading term in $1/\mathcal{Q}^2$ is picked up by the contraction with the longitudinal polarization vector ε_L. A straightforward leading twist calculation gives for the amplitude

$$\mathcal{A}_\mu^{\pi^+} = \sqrt{4\pi\alpha_{\rm em}} \frac{\pi}{N_c} \frac{f_\pi}{q^2} j_\mu \int_0^1 du \int_{-1}^1 dx\, \phi_\pi(u) T_{ud}\left(u, x, \xi\right) \frac{q\cdot A^{ud}}{q\cdot P}(x, \xi, \Delta^2) + \mathcal{O}\left(1/q^3\right), \tag{4}$$

where $q \equiv \frac{1}{2}(q_1 + q_2)$, $P = P_1 + P_2$ and $\Delta = q_1 - q_2 = P_2 - P_1$. The scaling variable is a generalized Bjorken variable $\xi = -\frac{q^2}{q\cdot P} \approx -\frac{q\cdot\Delta}{q\cdot P}$. Here in the last equality we neglected the pion mass, $m_\pi = 0$, and power suppressed corrections in the nucleon mass $M^2/(-q^2)$. The Lorentz structure $j_\mu = 2q_\mu + 3\xi P_\mu$, appearing in the twist-two part of $\mathcal{A}_\mu^{\pi^+}$, is gauge invariant to the twist-four accuracy, i.e. $q_{1\mu}\mathcal{A}_\mu^{\pi^+} = \left(q + \frac{\Delta}{2}\right)_\mu \mathcal{A}_\mu^{\pi^+} \approx 0$.

We introduced in Eq. (4) two non-perturbative objects, the pion distribution amplitude

$$\langle \pi^+(q_2) | \bar u(y) \gamma_\rho \gamma_5 d(z) | 0 \rangle = -iq_{2\rho} f_\pi \int_0^1 du\, \mathrm{e}^{iuq_2\cdot y + i(1-u)q_2\cdot z} \phi_\pi(u), \tag{5}$$

with the decay constant $f_\pi \approx 132$ MeV, as well as A_ρ^{ud} expressed in terms of the off-forward matrix element of the non-local flavour-changing operator

$$\langle n(P_2) | \bar d(y) \gamma_\rho \gamma_5 u(z) | p(P_1) \rangle = \int_{-1}^1 dx\, \mathrm{e}^{\frac{i}{2}(\Delta + xP)\cdot y + \frac{i}{2}(\Delta - xP)\cdot z} A_\rho^{ud}(x, \xi, \Delta^2). \tag{6}$$

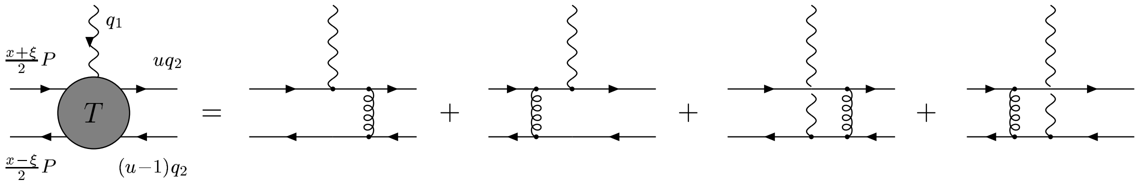

Figure 2: Hard-scattering coefficient function at leading order.

It gives rise to flavour non-diagonal GPDs, which enter as form factors

$$A^{ud}_\rho(x,\xi,\Delta^2) = \widetilde{h}_\rho \widetilde{H}^{ud}(x,\xi,\Delta^2) + \widetilde{e}_\rho \widetilde{E}^{ud}(x,\xi,\Delta^2), \tag{7}$$

in front of the Dirac structures

$$\widetilde{h}_\rho = \bar{U}_n(P_2)\gamma_\rho\gamma_5 U_p(P_1), \qquad \widetilde{e}_\rho = \frac{\Delta_\rho}{2M}\bar{U}_n(P_2)\gamma_5 U_p(P_1). \tag{8}$$

We can safely neglect the mass difference of proton and neutron and set in the following $M \equiv M_p = M_n$. Using the isospin symmetry of strong interactions, one can reduce the above GPDs to the conventional flavour diagonal proton matrix elements [4]

$$A^{ud}_\rho = A^{uu}_\rho - A^{dd}_\rho. \tag{9}$$

Evaluating the tree diagrams presented in Fig. 2, we get the known result for the hard amplitude T_{ud} to leading order (LO) accuracy in the QCD coupling [5, 6, 7],

$$T_{ud}(u,x,\xi) = C_F\alpha_s\frac{1}{\xi}\left\{\frac{Q_u}{(1-u)\left(1-\frac{x}{\xi}-i0\right)} - \frac{Q_d}{u\left(1+\frac{x}{\xi}-i0\right)}\right\} + \mathcal{O}(\alpha_s^2). \tag{10}$$

Here the quark charges are $Q_u = 2/3$ and $Q_d = -1/3$.

Let us remark on similarities of the present consideration to the one of the pion form factor since we will use them below. Contributing diagrams can be decomposed into two sets, where in the first (second) one the photon is attached to the (anti-) quark line. The momentum of the initial and final (anti-) quark is given in the collinear approximation by $p_1 = -\frac{x+\xi}{2\xi}\Delta$ $\left(p_1' = -\frac{x-\xi}{2\xi}\Delta\right)$ and $p_2 = uq_2$ ($p_2' = (1-u)q_2$), respectively. In leading twist approximation both of these sets separately respect current conservation. Obviously, if we formally replace $\frac{x+\xi}{2\xi} \to v$ (then $\frac{x-\xi}{2\xi} \to 1-v$) with $0 < v < 1$, we reduce the result (10) to the kinematics of the pion form factor. Of course, we can also recover the amplitude (10) from the pion form factor result by the substitution $v \to \frac{x+\xi}{2\xi}$. In addition we have to keep track of the emerging imaginary part, which is absent in the pion form factor. It develops now in the region $|x| \geq \xi$ and can be easily restored. Using this analogy, the NLO corrections of T_{ud} will be evaluated in section 3.

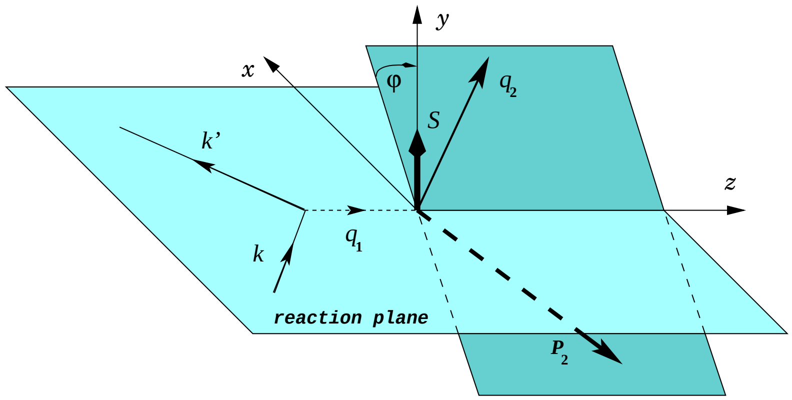

Figure 3: The reference frame for the cross section is defined as a target rest-frame and the z-axis directed along the three-vector of the virtual photon. The outgoing hadrons, pion and neutron, are in the same plane (due to the momentum conservation), which form an angle φ with the transverse polarization vector $\vec{S}$ of the target.

Now we turn to the calculation of the cross section for electroproduction of pions from the proton target $\ell(k)p(P_1) \to \ell(k')n(P_2)\pi^+(q_2)$. It is given by

$$d\sigma^{\pi^+} = \frac{1}{4k \cdot P_1} \left| L_\mu \mathcal{A}_\mu^{\pi^+} \right|^2 d\mathrm{LIPS}_3, \tag{11}$$

where the hadronic amplitude (4) is contracted with the leptonic current (which includes the photon propagator)

$$L_\mu = \frac{i}{\mathcal{Q}^2} \sqrt{4\pi\alpha_{\mathrm{em}}}\, \bar{u}(k')\gamma_\mu u(k), \tag{12}$$

and the three-particle phase space volume reads

$$d\mathrm{LIPS}_3 = (2\pi)^4 \delta^{(4)}(k + P_1 - k' - P_2 - q_2) \frac{d^3k'}{2E'(2\pi)^3} \frac{d^3P_2}{2E_2(2\pi)^3} \frac{d^3q_2}{2\varepsilon_2(2\pi)^3}. \tag{13}$$

In the rest-frame of the target with the z-axis chosen along the momentum of virtual photon, as shown in Fig. 3, we get the following four-fold cross section

$$\frac{d\sigma^{\pi^+}}{d\mathcal{Q}^2 dx_{\mathrm{B}} d|\Delta^2| d\varphi} = \frac{1}{2(4\pi)^4} \frac{x_{\mathrm{B}} y^2}{\mathcal{Q}^4} \left(1 + 4\frac{M^2 x_{\mathrm{B}}^2}{\mathcal{Q}^2}\right)^{-1/2} \left| L_\mu \mathcal{A}_\mu^{\pi^+} \right|^2, \tag{14}$$

where we use the variables $\mathcal{Q}^2 = -q_1^2$, $x_{\mathrm{B}} = -q_1^2/(2q_1 \cdot P_1)$, $y = q_1 \cdot P_1/k \cdot P_1$, and $\frac{\pi}{2} - \varphi$ is the azimuthal angle of the pion w.r.t. the reaction plane. They are related to the ones previously used by $q^2 \approx -\frac{1}{2}\mathcal{Q}^2$ and $\xi \approx x_{\mathrm{B}}/(2 - x_{\mathrm{B}})$.

A calculation of the matrix element squared results into

$$\frac{d\sigma^{\pi^+}}{d\mathcal{Q}^2 dx_{\mathrm{B}} d|\Delta^2| d\varphi} = \frac{\alpha_{\mathrm{em}}^2}{\mathcal{Q}^8} \frac{f_\pi^2}{N_c^2} \frac{x_{\mathrm{B}}(1-y)}{(2-x_{\mathrm{B}})^2} \Bigg\{ 8(1-x_{\mathrm{B}}) \widetilde{\mathcal{H}}^{ud\,*} \widetilde{\mathcal{H}}^{ud} - \frac{\Delta^2}{2M^2} x_{\mathrm{B}}^2 \widetilde{\mathcal{E}}^{ud\,*} \widetilde{\mathcal{E}}^{ud} - 4x_{\mathrm{B}}^2 \mathrm{Re}\left(\widetilde{\mathcal{H}}^{ud\,*} \widetilde{\mathcal{E}}^{ud}\right)$$
$$- 4x_{\mathrm{B}} \sqrt{1-x_{\mathrm{B}}} \sqrt{-\frac{\Delta^2}{M^2}} \sqrt{1 - \frac{\Delta_{\min}^2}{\Delta^2}} \sin\varphi\ \mathrm{Im}\left(\widetilde{\mathcal{H}}^{ud\,*} \widetilde{\mathcal{E}}^{ud}\right) \Bigg\}, \tag{15}$$

where we have chosen the transverse polarization for the target proton $S = (0, \cos\varPhi, \sin\varPhi, 0)$ along y-axis, $\varPhi = \pi/2$, so that it does not possess the longitudinal component in the laboratory frame with z-axis along the lepton beam. The azimuthal angle φ is between the proton spin and the projection of the pion momentum on the (x, y)-plane. For the minimal momentum transfer we use the approximation $\Delta^2_{\rm min} \approx -M^2 x_{\rm B}^2/(1 - x_{\rm B})$, which is valid for $x_{\rm B} M^2/\mathcal{Q}^2 \ll 1 - x_{\rm B}$. We also dropped kinematical $x_{\rm B}^2 M^2/\mathcal{Q}^2$ corrections in the cross section (15). The functions $\widetilde{\mathcal{H}}$ and $\widetilde{\mathcal{E}}$ are introduced as follows

$$\left\{ \begin{array}{c} \widetilde{\mathcal{H}}^{ud} \\ \widetilde{\mathcal{E}}^{ud} \end{array} \right\} (\xi, \Delta^2) = \int_0^1 du \int_{-1}^1 dx \; \phi_\pi(u) T_{ud}\,(u, x, \xi) \left\{ \begin{array}{c} \widetilde{H}^{ud} \\ \widetilde{E}^{ud} \end{array} \right\} \left(x, \xi, \Delta^2 \right). \tag{16}$$

The conversion from leptoproduction to photoproduction, $d\sigma_L^{\pi^+}$, with longitudinally polarized photons ε_L is done by multiplication of the result (15) by kinematical factors, namely,

$$d\sigma_L^{\pi^+} = d\sigma^{\pi^+} \left(\frac{|\varepsilon_L \cdot j|^2}{q_1 \cdot P_1} \right) \left(\frac{|L \cdot j|^2}{k \cdot P_1} \frac{d^3 k'}{2E'(2\pi)^3} \right)^{-1} = d\sigma^{\pi^+} \frac{1}{\alpha_{\rm em}} \frac{\pi}{1 - y} \frac{x_{\rm B}}{dx_{\rm B}} \frac{\mathcal{Q}^2}{d\mathcal{Q}^2}. \tag{17}$$

Here, to get the last equality we used $|\varepsilon_L \cdot j|^2 = 4\mathcal{Q}^2$, $|L \cdot j|^2 = 16(1 - y)/y^2$, $d^3k'/(2E') = \pi y/(2x_{\rm B})\, dx_{\rm B} d\mathcal{Q}^2$ and the known definition of y resulting from the ratio of the flux factors.

3 Next-to-leading order corrections.

Let us turn to the evaluation of the NLO corrections to the hard-scattering amplitude. As we noted above in section 2, the hard-scattering amplitude T_{ud} can be deduced from that one computed in the perturbative approach to the pion form factor[2] by the replacement $v \to \frac{x+\xi}{2\xi}$ and restoration of the imaginary part according to the causal Feynman prescription. Thus, we decompose the amplitude as

$$\begin{aligned} &T_{ud}\,(u, x, \xi \,|\mathcal{Q}, \mu_F, \mu_R) \qquad\qquad (18)\\ &\quad = C_F \alpha_s(\mu_R) \frac{1}{2\xi} \left\{ Q_u T \left(\frac{\xi - x}{2\xi} - i0, 1 - u \middle| \mathcal{Q}, \mu_F, \mu_R \right) - Q_d T \left(\frac{\xi + x}{2\xi} - i0, u \middle| \mathcal{Q}, \mu_F, \mu_R \right) \right\}, \end{aligned}$$

where

$$T\,(v, u\,|\mathcal{Q}, \mu_F, \mu_R) = \frac{1}{vu} \left\{ 1 + \frac{\alpha_s(\mu_R)}{2\pi} T^{(1)}\,(v, u\,|\mathcal{Q}, \mu_F, \mu_R) + \mathcal{O}\left(\alpha_s^2\right) \right\}. \tag{19}$$

Note that due to this $i0$-prescription, T_{ud} satisfy an unsubtracted dispersion relation that is consistent with that one for the $\gamma_L p \to \pi^+ n$ amplitude [1]. Analogous to the pion form factor we

[2] One has to use those results where the symmetry properties of the distribution amplitudes, the hard-scattering subprocess is convoluted with, was not used.

extracted the LO coefficient function from the NLO amplitude $T^{(1)}$. In our consequent presentation we will drop for simplicity the dimensionfull arguments, i.e. $\mathcal{Q}$, the factorization scale μ_F and the renormalization scale μ_R.

The one-loop correction $T^{(1)}$ for the pion form factor has been calculated by different authors in dimensional regularization, however, different renormalization and factorization prescriptions were applied [8, 9, 10, 11, 12, 13]. The occurring differences were clarified in Refs. [11, 12]. Indeed, if one takes into account errors, which are pointed out in Ref. [12], the results given in Refs. [9, 12] can be transformed to those in Refs. [8, 10, 11, 13]. They are given in the $\overline{\mathrm{MS}}$ scheme and the subtractions of infinities are done in a way that respects the universality of the distribution amplitude and the running coupling in this scheme. Thus, we have

$$T^{(1)} = C_F T^F + \beta_0 T^\beta + \left(C_F - \frac{C_A}{2}\right) T^{FA}, \tag{20}$$

with

$$\begin{aligned}
T^F &= [3 + \ln(vu)] \ln\left(\frac{\mathcal{Q}^2}{\mu_F^2}\right) + \frac{1}{2}\ln^2(vu) + 3\ln(vu) - \frac{\ln v}{2(1-v)} - \frac{\ln u}{2(1-u)} - \frac{14}{3}, \\
T^\beta &= \frac{1}{2}\ln\left(\frac{\mathcal{Q}^2}{\mu_R^2}\right) + \frac{1}{2}\ln(vu) - \frac{5}{6}, \\
T^{FA} &= \mathrm{Li}_2(1-v) - \mathrm{Li}_2(v) + \ln(1-v)\ln\left(\frac{u}{1-u}\right) - \frac{5}{3} \\
&+ \frac{1}{(v-u)^2}\Bigg\{(v+u-2vu)\ln(1-v) + 2vu\ln(v) \\
&+ \frac{(1-v)v^2 + (1-u)u^2}{v-u}\left[\ln(1-v)\ln(u) - \mathrm{Li}_2(1-v) + \mathrm{Li}_2(v)\right]\Bigg\} + \{v \leftrightarrow u\},
\end{aligned} \tag{21}$$

where $\mathrm{Li}_2(u) = -\int_0^u \frac{dx}{x}\ln(1-x)$ is the Euler dilogarithm. The colour factors are conventionally defined by $C_A = N_c$, $C_F = \frac{N_c^2-1}{2N_c}$ and the first coefficient of the QCD beta function reads $\beta_0 = \frac{4}{3}T_F N_f - \frac{11}{3}N_c$.

4 Leading vs. next-to-leading order observables.

To provide estimates for the cross section and the transverse single spin asymmetry as well as to discuss the scale setting ambiguities we use the following models for the GPDs. For $\widetilde{H}^{ud}$ we assume a factorizable ansatz of (x,ξ) and Δ^2 dependence valid at small Δ^2. In terms of the double distribution $F(y,z)$, we have

$$\widetilde{H}^{ud}(x,\xi,\Delta^2) = G(\Delta^2)\int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz\,\delta(y+\xi z - x) F^{ud}(y,z). \tag{22}$$

Here the form factor has a dipole parametrization $G(\Delta^2) = (1 - \Delta^2/m_A^2)^{-2}$ with $m_A^2 = 0.9$ GeV^2 and unit normalization. The double distribution is modeled according to Ref. [14]

$$F(y,z) = \{\Delta q(y)\theta(y) - \Delta\bar{q}(-y)\theta(-y)\}\, \pi(|y|,z), \qquad \pi(y,z) = \frac{3}{4}\frac{(1-y)^2 - z^2}{(1-y)^3}. \tag{23}$$

Assuming an $SU(2)$ symmetric sea, i.e. $\Delta\bar{q}^{ud}(y) = \Delta\bar{u}(y) - \Delta\bar{d}(y) = 0$, we have $\Delta q^{ud}(y) = \Delta u_{\rm val}(y) - \Delta d_{\rm val}(y)$. For the evaluations given below we use GSA forward parton densities at $Q^2 = 4$ GeV^2 [16]. Note that the simple ansatz (22) for $\widetilde{H}^{ud}$ is chosen in such a way that constraints arising from the reduction to the forward limit and the sum rule for the lowest moment are satisfied.

For $\widetilde{E}^{ud}$ we adopt the pion pole-dominated ansatz

$$\widetilde{E}^{ud}(x,\xi,\Delta^2) = F_\pi(\Delta^2)\frac{\theta\,(\xi > |x|)}{2\xi}\phi_\pi\!\left(\frac{x+\xi}{2\xi}\right), \tag{24}$$

with $F_\pi(\Delta^2)$ taken in our estimates in the form given in Eq. (39) of Ref. [15]. In the vicinity of the pion pole this form factor reads $F_\pi(\Delta^2 \to m_\pi^2) = 4g_A M^2/\,(m_\pi^2 - \Delta^2)$. For the pion distribution amplitude ϕ_π we will take for simplicity its asymptotic form[3]

$$\phi_\pi(u) = \phi^{\rm asy}(u) \equiv 6u(1-u), \qquad 0 \le u \le 1. \tag{25}$$

In the following we will discard the evolution effects of the GPDs for simplicity since they are small for the models and initial scale we chose. Especially, they are negligible for the asymptotic pion distribution amplitude since they arise solely from two-loop (and higher) effects. Same applies to $\widetilde{E}$ since it is proportional to the asymptotic distribution amplitude, too, i.e. $\xi\widetilde{E}^{ud}(x,\xi) \propto \phi_\pi\left((x+\xi)/(2\xi)\right)$. The neglected effects are of order 1% or so [17]. For $\widetilde{H}$ the evolution is more prominent since it shows up already at LO. However, it is still small for the change of scale from 4 GeV^2 to 10 GeV^2 and result in a $5-8\%$ change in the shape [17].

Let us now discuss the scale setting issues in the NLO coefficient function. A truncation of the perturbative series at a given order of coupling, here at NLO, causes a residual dependence on the factorization and renormalization scales. Obviously, this fact implies scale setting ambiguities, which result into uncertainties for the theoretical predictions. There are different suggestions to solve the scale setting problem with the aim to minimize the unknown higher order corrections. Unfortunately, they can provide quite different theoretical predictions for observables.

We start with the discussion of the μ_F dependence, which is intimately related to the evolution of the distribution amplitudes/GPDs. Note that this dependence completely disappears in $\widetilde{\mathcal{E}}$, if we

[3] It results into a good agreement of the theoretical predictions for the photon-to-pion transition form factor with experimental data. However, one should be aware that other quite different distribution amplitudes are consistent with the data (see for instance [18]).

assume (as we do) that ϕ_π coincides with the asymptotic distribution amplitude and one neglects its NLO evolution. The coefficient of $\ln(\mathcal{Q}^2/\mu_F^2)$ in Eq. (21) is merely the LO evolution kernel (convoluted with the LO coefficient function) and the asymptotic distribution amplitude is its eigenfunction with zero eigenvalue, e.g.

$$\int_{-1}^{1} dx\ \widetilde{E}^{ud}(x,\xi)\left(3+2\ln\frac{x\pm\xi}{2\xi}\right)=0.$$

In $\widetilde{\mathcal{H}}$ the term proportional to $\ln(\mathcal{Q}^2/\mu_F^2)$ survives now in the convolution with $\widetilde{H}$. However, as we pointed out above the evolution at LO is weak and the NLO corrections are small [17]. This implies that the μ_F-dependence in the $\widetilde{\mathcal{H}}$ amplitude is very weak. Therefore, we simply set $\mu_F=\mathcal{Q}$ in what follows. A detailed discussion of other plausible settings for μ_F runs beyond the scope of the present paper.

Next we turn to the μ_R scale. The variation of the amplitude with this scale appears at LO from the change of $\alpha_s(\mu_R)$. At NLO this scale ambiguity is expected to be smaller since this change is partially canceled by the variation of the term proportional to β_0 in Eq. (21). For reasons explained below, we will concentrate here only on two possibilities:

- The renormalization scale is set equal $\mathcal{Q}$, $\mu_R=\mathcal{Q}$.
- The Brodsky-Lepage-Mackenzie (BLM) scale setting prescription [19].

For the naive setting we observe in both $\widetilde{\mathcal{H}}$, separately for its real and imaginary part, and in $\widetilde{\mathcal{E}}$ function a cancelation of two large contributions: a positive one proportional to $(-\beta_0)$ and a negative one proportional to C_F. The term proportional to $(C_F-C_A/2)$ in Eq. (20) is not numerically important since it is relatively suppressed by $1/N_c^2$. Taken together they result for $N_f=3$ into a large positive effect.

The naive scale settings may not be quite appropriate by different reasons. The renormalization and factorization scales reflect quite different types of perturbative corrections [4]. The first one arises from the renormalization of ultraviolet divergences and generates the running of α_s, while the second one comes from the factorization of collinear singularities and provides the evolution of the distribution amplitudes and GPDs. The former originates from the geometric series with expansion terms having definite sign. On the other hand, the latter is of the evolution and remnant Sudakov type effects [20]. The exponentiated Sudakov corrections have a sign alternating expansion. Thus, while both partially cancel at NLO, it is expected that they will amplify at NNLO. Furthermore, since the hard external scattering scale is partitioned among a number of parton participants in the hard-scattering, their resulting virtuality is much softer than the external hard scale. Thus,

[4]In the consequent discussion we follow analogous considerations for the pion form factor in Ref. [20]

the coupling in the quark-gluon emission vertex has to be taken at a mean virtuality of e.g. hard gluon. A generalization of this idea results into a sensible prescription to absorb all effects coming from the running of the coupling, i.e. terms proportional to the β-function, into the scale of the coupling. This is the BLM procedure [19]. Since the trace anomaly of the energy momentum tensor is proportional to the β-function, the resulting series in α_s, if using this procedure, formally coincides with the perturbation theory in the conformally invariant (massless) QCD.

Since in general we have to deal with two different amplitudes defined in Eq. (16), which contain a real and imaginary part, the consequent application of the BLM setting procedure requires the introduction of different scales. We separately determine the scales for the real and imaginary parts of the functions $\widetilde{\mathcal{H}}$ and $\widetilde{\mathcal{E}}$ from the conditions

$$\int_0^1 \frac{du}{u} \int_{-1}^1 dx\ \phi_\pi(u) \left\{ Q_u \frac{T^\beta\left(\frac{\xi-x}{2\xi} - i0, u\right)}{\xi - x - i0} - Q_d \frac{T^\beta\left(\frac{\xi+x}{2\xi} - i0, u\right)}{\xi + x - i0} \right\} \left\{ \begin{array}{c} \widetilde{H}^{ud} \\ \widetilde{E}^{ud} \end{array} \right\} \left(x, \xi, \Delta^2\right) = 0. \quad (26)$$

Note that the scales will depend on the shape of the distribution amplitude and the GPDs.

For $\widetilde{\mathcal{E}}$ there is no imaginary part since $\widetilde{E}$, as defined in Eq. (24), is concentrated solely in the exclusive domain. For its real part due to $x \to -x$ symmetry both the Q_u and Q_d contributions in Eq. (18) are the same and one immediately finds that the BLM scale

$$\mu_R^2 = \mathcal{Q}^2 \mathrm{e}^{-14/3} \quad (27)$$

coincides with the one of the pion form factor [21, 20] for the asymptotic distribution amplitude.

To discuss the scale setting in $\widetilde{\mathcal{H}}$, let us first consider the properties of $\widetilde{H}$. From Eqs. (22,23) with $\Delta\bar{q} = 0$ it is obvious that the function vanishes for $x < -\xi$, $\widetilde{H}^{ud}(x < -\xi, \xi) = 0$. From this we conclude that the imaginary part of Q_d contribution vanishes. As we mentioned above, for $\widetilde{H}$ the scale will be different for the imaginary and real parts and will depend on the skewedness

$$\mu_R^2 = \mathcal{Q}^2 \mathrm{e}^{-f(\xi)}. \quad (28)$$

For the imaginary part one gets

$$f_{\mathrm{Im}}(\xi) = \frac{19}{6} - \ln\frac{1-\xi}{2\xi} - \int_\xi^1 dx\ \frac{1 - \widetilde{H}^{ud}(x,\xi)/\widetilde{H}^{ud}(\xi,\xi)}{\xi - x}. \quad (29)$$

The scale for the real part of the $\widetilde{\mathcal{H}}$-contribution is governed by the function

$$f_{\mathrm{Re}}(\xi) = \frac{19}{6} - \frac{\int_{-1}^1 dx\ \left\{ Q_u \frac{\mathrm{PV}}{\xi-x} \ln\frac{|\xi-x|}{2\xi} - Q_d \frac{\mathrm{PV}}{\xi+x} \ln\frac{|\xi+x|}{2\xi} + \frac{\pi^2}{2}\left(Q_u\delta(\xi-x) - Q_d\delta(\xi+x)\right)\right\} \widetilde{H}^{ud}(x,\xi)}{\int_{-1}^1 dx\ \left\{ Q_u \frac{\mathrm{PV}}{\xi-x} - Q_d \frac{\mathrm{PV}}{\xi+x}\right\} \widetilde{H}^{ud}(x,\xi)}, \quad (30)$$

where the symbol PV stands here for the Cauchy principal value prescription.

For the models we are using in all of the three cases e^{-f} is below 0.1 in a wide interval of Bjorken variable $0.1 < x_B < 0.5$. This means that the argument of the coupling is driven into the infrared region. There are experimental indications that the coupling is frozen at such a low scale [22]. Indeed, in exclusive processes, such as nucleon form factors, fixed angle proton-proton elastic scattering etc., the data at large scales are consistent with the predictions of dimensional counting rules. On the other hand, the perturbative leading twist QCD analyses coincides with the dimensional counting rules, however, is proportional to powers of α_s, e.g. α_s^2/Q^4 and α_s^6/t^8, respectively. Since higher order analyses favour a low scale in the coupling, we may conclude that the latter is a slowly varying function in this domain [22]. For our purposes we choose a frozen coupling with $\alpha_s(\mu_R^2)/\pi = 0.1$ for $\mu_R^2 < 1$ GeV^2 [21, 20] instead of using three different soft scales in the analytical coupling, e.g. with gluon mass $\mu_R^2 \to \mu_R^2 + 4m_g^2$.

Now we are in a position to present numerical estimates of the observables at LO and NLO. Due to the asymptotic form of the distribution amplitude, which we use, the integration w.r.t. the momentum fraction u can be done analytically:

$$\begin{aligned} \mathcal{T}_{ud}(x,\xi) &= \int_0^1 du\, \phi^{\rm asy}(u) T_{ud}(u,x,\xi) \\ &= C_F \alpha_s(\mu_R) \frac{1}{2\xi} \left\{ Q_u \mathcal{T}\left(\frac{\xi - x}{2\xi} - i0 \right) - Q_d \mathcal{T}\left(\frac{\xi + x}{2\xi} - i0 \right) \right\}, \end{aligned} \tag{31}$$

where

$$\begin{aligned} \mathcal{T}(v) &= \int_0^1 du\, \phi^{\rm asy}(u) T(v,u) \\ &= \frac{1}{v} \left\{ 3 + \frac{\alpha_s(\mu_R)}{2\pi} \left(C_F \mathcal{T}^F(v) + \beta_0 \mathcal{T}^\beta(v) + \left(C_F - \frac{C_A}{2} \right) \mathcal{T}^{FA}(v) \right) + \mathcal{O}\left(\alpha_s^2\right) \right\}, \end{aligned} \tag{32}$$

with

$$\begin{aligned} \mathcal{T}^F &= \frac{3}{2}[3 + 2\ln v] \ln\left(\frac{\mathcal{Q}^2}{\mu_F^2} \right) + \frac{3}{2} \ln v\, [3 + \ln v] - \frac{3 \ln v}{2(1-v)} - \frac{77}{4}, \\ \mathcal{T}^\beta &= \frac{3}{2} \ln\left(\frac{\mathcal{Q}^2}{\mu_R^2} \right) + \frac{3}{2} \ln v - \frac{19}{4}, \\ \mathcal{T}^{FA} &= -1 - 6[\ln v + 2\ln(1-v)] + 12(1-3v) \left\{ \mathrm{Li}_2(v) - \mathrm{Li}_2(1-v) + \ln\frac{v}{1-v} \right\} \\ &\quad + 6(1 - 6v + 6v^2) \left\{ 3\mathrm{Li}_3(v) + 3\mathrm{Li}_3(1-v) - \ln\frac{v}{1-v}\, \mathrm{Li}_2(v) + \ln^2(1-v) \ln v \right. \\ &\quad \left. - \frac{\pi^2}{6}[\ln v + 2\ln(1-v)] \right\}, \end{aligned} \tag{33}$$

and $\mathrm{Li}_3(v) = \int_0^v du \mathrm{Li}_2(u)/u$.

The LO predictions were done for the running coupling with $\mu_R = \mathcal{Q}$, $\Lambda_{\rm QCD}^{\rm LO} = 220$ MeV and $N_f = 3$. At NLO we use, as discussed above, two scale setting procedures: the naive $\mu_R = \mathcal{Q}$

and BLM one, with the running $\alpha_s(\mathcal{Q}^2)$ in the first and a fixed $\alpha_s/\pi = 0.1$ below 1 GeV^2 in the second case, respectively. Our LO predictions given in Fig. 4 (a, b) are in agreement with recent analyses in Refs. [5, 6, 7]. However, they are plagued by large uncertainties from the higher order corrections. It is interesting to note that taking forward parton distribution as a model for the GPD, one gets almost identical results for the cross section.

The transverse single spin asymmetry defined by

$$A_\perp = \left(\int_0^\pi d\varphi \frac{d\sigma_L^{\pi^+}}{d|\Delta^2| d\varphi} - \int_\pi^{2\pi} d\varphi \frac{d\sigma_L^{\pi^+}}{d|\Delta^2| d\varphi} \right) \left(\int_0^{2\pi} d\varphi \frac{d\sigma_L^{\pi^+}}{d|\Delta^2| d\varphi} \right)^{-1} \tag{34}$$

is studied numerically in Fig. 4 (c, d) for $\Delta^2 = -(0.1,\ 0.3)$ GeV^2. Our leading predictions shows the same large asymmetry as has originally been observed in Ref. [6]. As our studies demonstrate this observable is less sensitive to higher order effects.

5 Discussion and conclusions.

We have discussed in the present Letter the cross section and transverse proton single spin asymmetry in the hard exclusive production of pions. For the longitudinal polarization of the virtual photon the amplitude is dominated by twist-two contributions, i.e. lowest Fock components in the pion and $p \to n$ transition amplitudes. We evaluated the NLO corrections to the hard parton subprocess basing on the available result for the pion form factor and estimated their size in physical observables. We found that the ambiguity in the renormalization scale setting (together with the freezing of the coupling in the infrared region) results into large $^{+40\%}_{-70\%}$ uncertainties in the theoretical predictions, $(T_{NLO} - T_{LO})/T_{LO}$, of the amplitude. After application of the BLM scale setting prescription, we observed a large reduction in the magnitude of the cross section related to the effects of Sudakov double logarithms. A deeper insight into the structure of these corrections is highly desirable.

The theoretical uncertainty in the factorization procedure on the amplitude level is translated into large variations of the physical cross section. However, we found that the single spin asymmetry, given by the ratio of the Fourier coefficients of the cross section, is a 'good' observable since the ambiguities due to the truncation of the perturbative series approximately cancel. Thus, the perturbative predictions for this quantity are rather stable. The NLO effects result into $^{+7\%}_{-18\%}$ corrections to the LO prediction for $0.1 < x_{\rm B} < 0.5$. For the assumed models of the non-perturbative functions the asymmetry is big, being as large as 60%, and is sensitive to the pion pole dominated GPD $\widetilde{E}$. Thus, in view of its advantages being rather insensitive to the higher order corrections, it turns out to be an appropriate quantity for experimental studies at Jefferson Lab and HERMES. The large NLO corrections may indicate that the application of the perturbative approach to the

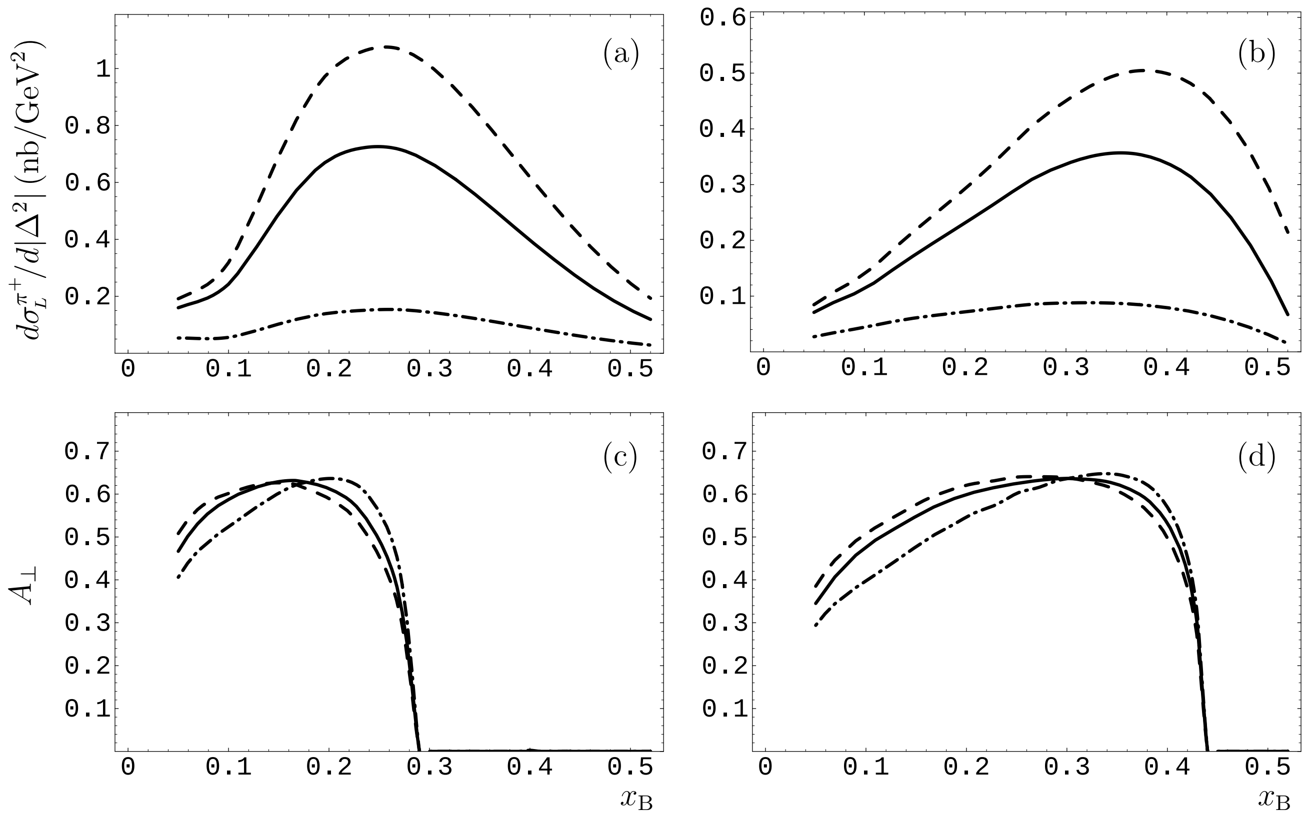

Figure 4: The leading twist predictions for the unpolarized photoproduction cross section $d\sigma_L^{\pi^+}/d|\Delta^2|$ at $\mathcal{Q} = 10$ GeV^2 for the GPD models specified in Eqs. (22,24) are shown for $\Delta^2 = \Delta^2_{\rm min}$ and $\Delta^2 = -0.3$ GeV^2 in the panels (a) and (b), respectively. In (c) and (d) we display the transverse proton single spin asymmetry $A_\perp$ for $\Delta^2 = -0.1$ GeV^2 and $\Delta^2 = -0.3$ GeV^2, respectively. The solid, dashed and dash-dotted curves represent the LO and NLO with the naive and BLM scale setting, respectively.

meson production cross section is legitimate at a rather large momentum transfer. However, for the asymmetry, which depends at leading twist only logarithmically on $\mathcal{Q}^2$, one expects its earlier validity due to an essential cancelation of higher order perturbative and, hopefully, also power suppressed corrections, due to their intrinsic interrelation in field theoretical treatment, recall renormalons in this respect.

Let us stress that the longitudinal proton single spin asymmetry, measured at HERMES [2], arises at the twist-three level and requires a separate analysis. The result we have presented here are given for the transversely polarized proton target. The NLO analysis addressed in our study can be extended to a large set of meson leptoproduction amplitudes with the quark dominated short-distance subprocess, e.g. $\ell p \to \ell' \pi^0 p$, $\ell p \to \ell' \rho^+ n$, etc.

We would like to thank J.C. Collins, P. Kroll, J. Smith, M. Stratmann for useful conversations

and especially A.V. Radyushkin for illuminating and instructive discussions. D.M. is grateful to J. Smith for the hospitality extended to him at the C.N. Yang Institute for Theoretical Physics where a major part of this work has been written.

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